Exam J (Part II) **Name**

Please Note: Calculators may be used only in elementary computational and trig modes, and not in calculus mode (even for exploratory purposes). Please note that points will be deducted for solutions that are not legibly expressed and clearly organized. Suggestion: look to the list of mathematical expressions at the end of the exam for hints and inspiration.

Solve the first 5 problems on the extra sheets of paper provided. Then place your answers in the corresponding spaces below. Partial Credit is possible.

1. Consider the function $f(x) = \sqrt{|x|}$.

The quantity $f'(-2)$ expressed as a limit is ____________________________ .

The numerical value of $f'(-2)$ is ____________ .

2. The limit $\lim\limits_{\Delta x \to 0} \frac{\tan^{-1}(\ln(1+\Delta x))}{\Delta x}$ is the derivative of a certain function for a certain value of x. The function is ____________________ and the value of x is ____________ .

The value of the limit is ____________________ .

3. Consider the function $f(x) = \frac{x^2}{e^x}$. Its derivative is $f'(x) = \frac{2x-x^2}{e^x}$. The x coordinates of the points where the graph has horizontal or vertical tangents are

$x =$ ____ (identify whether horizontal or vertical in each case). The coordinates of the horizontal or vertical asymptotes are

________________________ (identify whether horizontal or vertical in each case). The interval(s) of increase of the graph are ________________________________ and the interval(s) of decrease are ________________________________ (identify which are intervals of increase and which of decrease).

4. The second derivative of the function of Problem 3 is $f''(x) = \frac{x^2-4x+2}{e^x}$. The interval over which the graph of the function is concave down is ____________________ and the intervals over which the graph is concave up are _________________________ . The graph has points of inflection at ____________________ .

5. Let $f(x) = x^{\sin x}$. The derivative of $f(x)$ at $x = \frac{\pi}{2}$ is ____________________ .

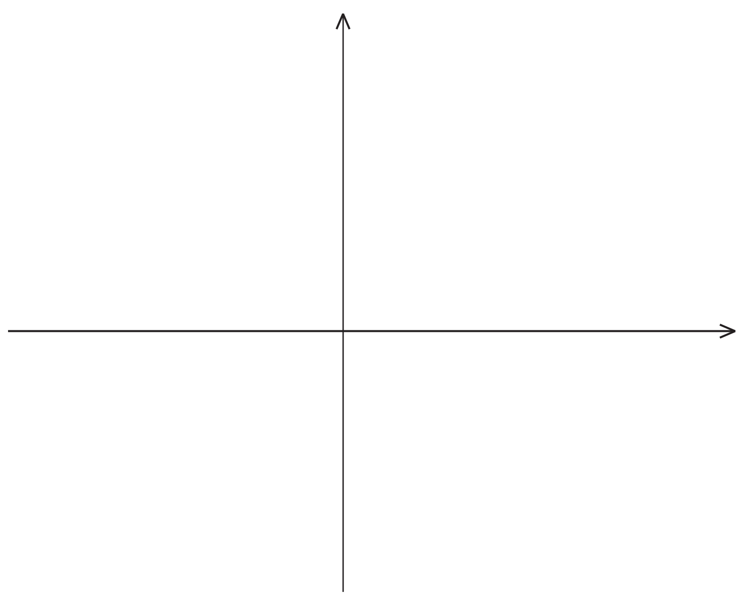
Comment about $f'(0)$: ____

For the solutions of the rest of the problems, supply the details in a clear and well organized way.

6. A double use of L'Hospital's rule provides the equalities $\lim_{x\to 0} \frac{x}{x^2} = \lim_{x\to 0} \frac{1}{2x} = \lim_{x\to 0} \frac{0}{2} = 0$. Is this correct or not? If not, what's the problem?

7. A function $y = f(x)$ is known to satisfy $f(0) = 0$, $f'(0) = 1$, and $f''(x) = -f(x)$. Determine the only function that satisfies these three conditions.

8. Consider the polar function $r = f(\theta) = 4\sin\theta$. Determine the Cartesian equivalent of $r = 4\sin\theta$ and use it to sketch the graph of $r = f(\theta) = 4\sin\theta$ in the space provided below.



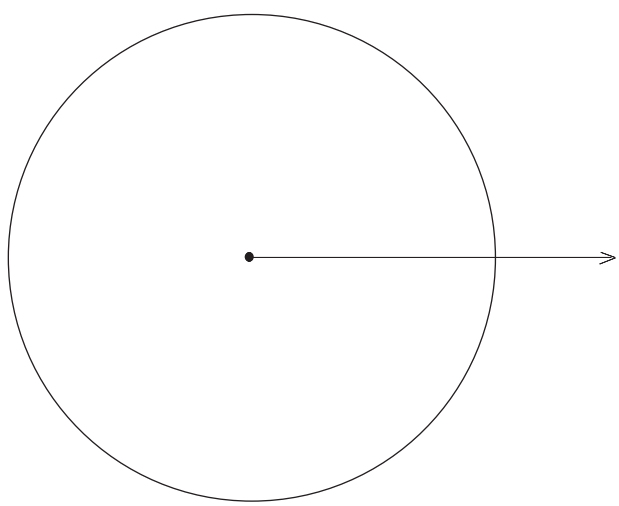
Explain what the integrals $\int_0^{\frac{\pi}{2}} \frac{1}{2} f(\theta)^2\, d\theta$ and $\int_0^{\frac{\pi}{2}} \sqrt{f'(\theta)^2 + f(\theta)^2}\, d\theta$ mean and determine their values by using the graph.

9. Consider a plane with a polar coordinate system. The unit of length is the centimeter. A line starting at the origin O lies on top of the polar axis. A small bug sits at O. At time $t = 0$ the line begins to rotate counter clockwise (while remaining fixed at O) at a rate of π radians per minute. Also at time $t = 0$ the bug begins to crawl along the rotating line with a speed of 2 centimeters per minute.

i. Find the polar coordinates of the position of the bug at $t = 4$ minutes.

ii. Let $(r(t), \theta(t))$ be the position of the bug at any time t. Use information about $\theta'(t)$ and $r'(t)$ to find a function $r = f(\theta)$ such that the graph of this function describes the path of the moving bug.

iii. Graph the position of the bug from time $t = 0$ minutes to the time $t = 4$ minutes in the space below. The radius of the circle is 8 centimeters.



Formulas and Facts:

$A = \frac{1}{2} r^2 \theta$

$a^2 = b^2 + c^2 \qquad \varepsilon = \frac{c}{a} \quad A = ab\pi$

$F = ma,\ \kappa = \frac{A_t}{t}$

$\frac{d}{dx} a^x = \ln a \cdot a^x \ \log_a x = \frac{1}{\ln a} \cdot \ln x$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1}$

$x = r\cos\theta, \quad y = r\sin\theta, \quad r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$

$r(t)^2 \cdot \theta'(t) = c \qquad F(t) = m\left[\frac{4\kappa^2}{r(t)^3} - \frac{d^2r}{dt^2}\right]$

$x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt} \qquad 2\frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2} = 0$

$h''(\theta) + h(\theta) = 0 \qquad A\sin\theta + B\cos\theta$

$\int_a^b \sqrt{1 + f'(x)^2}\, dx \qquad \int_a^b \sqrt{f'(\theta)^2 + f(\theta)^2}\, d\theta$

$\int_a^b \frac{1}{2} f(\theta)^2\, d\theta \qquad f'(\theta) = f(\theta) \cdot \tan(\alpha - \frac{\pi}{2})$

$a = \frac{d}{1-\varepsilon^2}$ and $b = \frac{d}{\sqrt{1-\varepsilon^2}} \qquad a = \frac{d}{\varepsilon^2 - 1}$ and $b = \frac{d}{\sqrt{\varepsilon^2 - 1}}$